Particle decay processes, the quantum Zeno effect and the continuity of time

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Signal-state quantum mechanics is used to discuss quantum mechanical particle decay probabilities and the quantum Zeno effect. This approach avoids the assumption of continuous time, conserves total probability and requires neither non-Hermitian Hamiltonians nor the ad-hoc introduction of complex energies. The formalism is applied to single channel decays, the ammonium molecule, and neutral Kaon decay processes.

I. INTRODUCTION

The nature of time is not generally regarded as one of the fundamental problems in quantum mechanics (QM). It is normally assumed to be a continuum. Temporal continuity is central to the formalism of classical mechanics (CM) and relativity, and many of the concepts derived from it have analogues in quantum theory. For example, the Hamiltonian is a generator of translation in time in both CM and QM. In standard quantum mechanics (SQM), the empirical success of the Schrödinger equation is usually taken as supporting temporal continuity, although there have been many attempts to construct discrete time quantum mechanics (Caldirola 1978, Yamamoto 1984, Bender et al. 1985, Golden 1992, Jaroszkiewicz and Norton 1997). Significantly, all successful quantum field theories such as QED and the standard model are based on continuous time. If temporal continuity were removed, the ramifications would be enormous in a number of disciplines.

On closer inspection, however, the continuity of time does not look quite so obvious. A problem emerges, involving a clash between two contrasting and essentially mutually exclusive observations. These were discussed in detail by Misra and Sudarshan (M&S) in an influential paper on the quantum Zeno effect (Misra and Sudarshan 1977). On the one hand, there is currently no known principle in SQM which forbids the continuity of time. Because of this, the axioms of SQM are invariably stated in terms of continuous time and usually include the Schrödinger equation explicitly (Peres 1993). On the other hand, there is the empirical fact that no experiment can ever monitor a system in a continuous way. The best that can be done is to perform a sequence of experiments with a decreasing measurement time scale, in an attempt to see evidence for temporal continuity, such as in the phenomenon known as the quantum Zeno effect (Bollinger et al. 1990).

The standard response to this problem is to assert that, because experiments *could in principle* be conducted with an arbitrarily small measurement time scale τ, there is in fact no barrier to taking the limit $\tau \rightarrow 0$. This argument is unsound because all the theoretical and empirical evidence actually points in the opposite direction. On the theoretical side, Caldirola (1978) proposed a model for the electron based on the existence of a fundamental time scale, the chronon, of the order 10^{-23} seconds. On a different front, data from high energy particle scattering experiments indicates that at a certain scale of energy, the weak, electromagnetic and strong interaction coupling constants run to a common limit, a phenomenon known as Grand Unification. The scale of energy involved corresponds to a timescale of about 10^{-33} seconds. When gravity is included in an attempt to unify all known interactions, the relevant timescale reduces to the Planck time, of the order 10^{-44} seconds. This is popularly regarded as representing the smallest meaningful temporal interval possible, although that belief itself has been questioned (Meschini 2006).

The Grand Unification and Planck timescales are generally regarded as most relevant to early universe cosmology, although there are ongoing attempts in high energy particle physics scattering experiments to push interaction energies towards the Grand Unification scale. These timescales however are far removed from any direct measurement timescales relevant to laboratory physics. The smallest unit of time that has been measured in the laboratory is on the attosecond (10^{-18}) timescale. Even if this were improved on, long before Grand Unification or Planck scales could be reached, the nucleonic timescale of the order 10^{-23} seconds (about the time for light to cross a proton and coincidentally about the same as Caldirola's chronon) would step in to provide a realistic temporal barrier to the continuum limit. This is almost certainly the shortest timescale that could be achieved in any conceivable experiment on the quantum Zeno effect. What characterizes quantum Zeno effect experiments is the need to probe a system repeatedly, in an attempt to simulate a continuum of observations. That is quite different to what happens in a high energy particle collision experiment, which can be regarded as a succession of one-off observations per run.

The conventional belief in temporal continuity, therefore, is no more than an article of faith, a temporal equivalent of Fourier's principle of similitude (Wigner, 1949), which is known to be false. This principle asserts that the physical properties of a system is independent of its scale, and is flatly contradicted by the existence of atoms and the finite speed of light. Temporal continuity is a metaphysical (i.e., unprovable) assumption best avoided

if at all possible, which is one of the principles on which our work is based.

M&S discussed a number of themes related to the nature of time. They analyzed particle decay processes and certain questions not normally discussed in SQM. Three of these questions were referred to as P, Q and R and this convention will be followed here. The question $P(0, t; \rho)$ asks for the probability that an unstable system prepared at time zero in state ρ has decayed sometime during the interval $[0, t]$; the question $Q(0, t; \rho)$ asks for the probability that the prepared state has not decayed during this interval; and finally, the question $R(0, t_1, t; \rho)$ asks for the probability that the state has not decayed during the interval $[0, t_1]$, where $0 < t_1 < t$, and has decayed during the interval $[t_1, t]$. M&S stressed that these questions are not what SQM normally calculates, which is the probability distribution of the time at which decay occurs, denoted by q. The difference here is that the P, Q and R questions involve a continuous set of observations, or the nearest practical equivalent of it, whereas q involves a set of repeated runs, each with a one-off observation at a different time to determine whether the particle has decayed or not at that time. Because P, Q and R involve a different experimental protocol to q, it should be expected that empirical differences will be observed. Note that the observations M&S refer to can have negative outcomes, i.e., an answer that a particle has *not* decayed by a certain time counts as an observation.

The points raised by M&S were used by them to emphasize the limitations of SQM. Although it works excellently in all known situations, SQM does not readily give a complete picture of experimental questions such as P, Q and R. M&S discussed a range of alternative resolutions to the questions raised, concluding that "there is no standard and detailed theory for the actual coupling between quantum systems and the *classical measuring apparatus*".

Our paper presents an approach to quantum mechanics, *signal-state quantum mechanics* [1](SSQM), which addresses some of the issues raised by M&S (Jaroszkiewicz and Ridgway-Taylor 2006, Jaroszkiewicz and Eakins 2006). In SSQM, quantum wave-functions are interpreted as probability amplitudes for signals obtained from physical apparatus by observers. This is in contrast to SQM, in which quantum wave-functions are generally assumed to describe the properties of systems under observation, such as electrons, photons, atoms or molecules.

SSQM was motivated greatly by Heisenberg's original vision of quantum mechanics, in which only quantities accessible to an observer are regarded as physically meaningful (Heisenberg 1927). Its mathematical structure has been designed to reflect the fact that experimentalists never deal directly with particles per se. Instead, they push buttons, look at screens and count signals, and by such means extract information from their apparatus. Everything else is inferred.

In SSQM, some concepts from quantum information theory are used to model states of an observer's apparatus, which is assumed to consist of a varying number of elementary detectors. Each of these has the potential to provide the observer with a yes/no answer to the basic question "*is there a signal here at this time?*" M&S touched upon this aspect by working with a projection operator E applied at a given time to an evolving unstable particle state, corresponding to the yes/no question "*is the particle in its undecayed state at this time?*". Crucially, M&S included the decay product states in their Hilbert space, and this also has an analogue in the SSQM approach.

An important feature in SSQM is that the observer need not choose to look at any given detector at any given time, and it is here that the quantum nature of an experiment manifests itself. At any given time, the observer's information about the potential state of the apparatus can be a non-classical superposition of alternative classical signal states, prior to any question being actually asked. Such a state will be called a *labstate*, to distinguish it from the concept of system state as used in SQM.

When one or more elementary questions are actually asked and answered, this inevitably requires the observer to physically interact with the corresponding detectors. The result is the extraction of physical information in the form of yes or no answers, and this necessarily modifies the observer's knowledge of the labstate. It is the labstate which can be said to collapse, rather than any wave-function of the system under observation. By placing emphasis on the apparatus and avoiding any focus on system states, SSQM avoids metaphysical speculation about the relationship between particles, waves and non-classical particle trajectories.

It is a basic premise in SSQM that all physics experiments can be formulated in terms of sufficiently many elementary questions and answers. All the standard concepts of SQM are to be found in SSQM but in a modified and usually enhanced form. There are some important additional concepts which arise because of the need to model laboratory physics more realistically than is the rule in SQM. Particle decay experiments provide an excellent forum to discuss these ideas, because such experiments are generally much more complicated in their space-time structure than the simple SQM description implies.

One of the consequences of this simplification is that when decay products are not included in the Hilbert space in SQM, it usually becomes necessary to introduce imaginary terms into effective Hamiltonians, in order to reflect the inevitable non-conservation of probability which arises in consequence. Crucially, M&S did not make this simplification and so their formalism did not invoke any such assumptions. However, this left them with significant mathematical problems, which they solved only by making conventional assumptions concerning evolution operators and the continuity of time. For example, they assumed that the standard evolution operator $U(t) \equiv \exp(-iHt/\hbar)$ is an element of a strongly continuous one-parameter family of unitary operators in a separable Hilbert space. In the SSQM formalism, there is a different Hilbert space associated with each time step,

so such assumptions do not have any natural place.

In the SSQM approach, total probability is always conserved. This is achieved through the use of an analogue of unitarity called *semi-unitarity*. This is not just a technical trick but an essential feature of the theory. It will be shown that semi-unitarity ensures that overall probability is conserved and that exponentially falling survival probabilities can arise in a natural way.

In SSQM, the number of qubits required to describe an experiment can change with time. At any given time, the qubits needed to describe the apparatus form what is called a *Heisenberg net*. This is discussed in detail in the next section. In particle decay processes, it is found that the size of the Heisenberg net required to describe such processes grows linearly with time, reflecting the basic fact that a particle decay experiment is an irreversible quantum process occurring over extended regions of space-time. In such processes, information can be extracted not only at the end of a run, but at any time between the start (state preparation) and the end.

The mathematical basis of SSQM will be reviewed briefly in the next section, followed by a discussion of how it can be used to describe the simplest idealized decay process, that of a particle decaying via one channel only. The quantum Zeno effect makes an appearance at this point, and it will be shown that there is no unambiguous answer as to whether a system decays whilst it is being monitored or whether it remains in its initial state.

One of the aims of this paper is to show how more complex phenomena such as neutral Kaon decay can be discussed in SSQM. To do this, it will be necessary to review the SSQM description of the ammonium molecule. The techniques developed there are then used to show how the Kaon decay regeneration amplitudes discussed by Gell-Mann and Pais arise naturally in the SSQM approach. It will be seen not to be necessary to introduce any ad-hoc imaginary terms in any amplitudes or to use non-Hermitian Hamiltonians.

II. SIGNAL-STATE QUANTUM MECHANICS

Regardless of how wave-functions are interpreted, all quantum experiments are exercises in information extraction. It has been argued above that this cannot be done in a truly continuous way, even in those experiments designed to demonstrate the quantum Zeno effect (Bollinger 1990) or support the predictions of decoherence theory (Zurek 2002). To model this fact, the formalism of SSQM works with a discrete concept of time rather than a continuum. What an observer says about their apparatus at any given discrete time will be labelled by an integer, with a value $n + 1$ denoting a stage later than that denoted by n. Each stage represents one of two things: either a definite change in information about the state of the apparatus, or else a change in the observer's quantum mechanically based prediction as to what the state of the apparatus should be at that time, if an observation were to be made. What these discrete times actually mean in terms of intervals of physical units of time can

be decided later. Moreover, there is no need to regarded these intervals as all equal.

The elementary detectors modelled in SSQM should not be regarded as necessarily localized in physical space, or as giving information about particle position only, although that is certainly possible and may be important in many situations. These detectors could be constructed from widely spaced physical components distributed around a laboratory, and associated with non-localized variables such as particle momenta, or with spin, or any other physically meaningful concept. What they all have in common is that they give only *yes* or *no* answers to the elementary questions "has this detector fired or not?"

A detector is perhaps best regarded as a physical process which can have one of two possible outcomes. Given this, it is natural to model such detectors by *bits* in classical physics and by quantum bits (qubits) in the quantum scenarios of interest here. The ith detector at time n will be modelled by the quantum bit $\mathcal{Q}_n^i$, which is a two dimensional Hilbert space. Such a qubit should not be identified with any dichotomous variable such as spin half angular momentum, and is neither a fermion nor a boson (Wu and Lidar 2002), although the mathematical formalism suggests the former occasionally.

At any given time n, the number r_n of detector qubits available to the observer *at that time* will be called the *rank* of the corresponding Heisenberg net $\mathcal{H}_n \equiv \mathcal{Q}_n^1 \otimes \mathcal{Q}_n^2 \otimes \ldots \otimes \mathcal{Q}_n^{r_n}$. This net is the tensor product of all the relevant qubits and is a Hilbert space of dimension $d_n \equiv 2^{r_n}$. In those scenarios involving large numbers of detectors, the associated Heisenberg nets will have much greater dimensions than the corresponding Hilbert space in SQM, but there is a sensible physical interpretation of this (Jaroszkiewicz and Ridgway-Taylor 2006).

Equally significantly, Heisenberg nets contain entangled states as well as separable states, which is a fundamental feature of quantum mechanics. In contrast to SQM, the SSQM view is that it is not system states which may be entangled but states of the apparatus (the lab-states). This does not mean that the apparatus itself is entangled. In conventional experiments, experimentalists generally know what their apparatus is [2]. What they do not know beforehand is how it will behave in quantum outcome terms.

Regarding entanglement as something to do with states of apparatus seems less objectionable and more natural than thinking of particles themselves as being entangled. From this point of view, entanglement is just a manifestation of how the *context* (which is a form of information) of an experiment influences a quantum process. For example, abstract four dimensional Hilbert space and the tensor product of two qubits are mathematically isomorphic spaces in many respects, but only the latter space contains entangled elements.

In SSQM there is a natural, preferred basis B_n for $\mathcal{H}_n$, consisting of all possible classical (i.e., sharp) signal states. These may be defined in terms of excitations of the *void state* $|0, n)$, the unique state in $\mathcal{H}_n$ for which every detector is in its "no" state, i.e., $|0, n) \equiv$

$|0, n)_1|0, n)_2 \ldots |0, n)_{r_n}$. Here and elsewhere the tensor product $\otimes$ symbol will be suppressed. The origin of this uniqueness arises from the knowledge that the observer has about their apparatus. A Heisenberg net is not just a tensor product space, but includes the observer's knowledge of what their apparatus means, and it is this which selects B_n.

To describe the preferred basis signal states, it is convenient to introduce a set $\{\mathbb{A}_{i,n}^+, \ i = 1, 2, \ldots, r_n\}$ of *signal operators* (Jaroszkiewicz and Eakins 2006). There is one signal operator $\mathbb{A}_{i,n}^+$ associated with each qubit $\mathcal{Q}_n^i$ in the Heisenberg net $\mathcal{H}_n$, with the property that it changes the void state $|0, n)_i$ in $\mathcal{Q}_n^i$ to the signal state $|1, n)_i$ and leaves all the other qubits unaffected, i.e.,

$$\mathbb{A}_{i,n}^+|0, n) = |0, n)_1 \ldots |0, n)_{i-1}|1, n)_i|0, n)_{i+1} \ldots |0, n)_{r_n}, \tag{1}$$

for $i = 1, 2, \ldots, r_n$. These signal operators are neither bosonic nor fermionic. They obey the rules

$$[\mathbb{A}_{i,n}^+, \mathbb{A}_{j,n}^+] = 0, \quad \{\mathbb{A}_{i,n}, \mathbb{A}_{i,n}^+\} = \mathbb{I}_n, \quad i = 1, 2, \ldots, r_n, \tag{2}$$

where $\mathbb{I}_n$ is the identity operator for $\mathcal{H}_n$, and the nilpotency condition

$$\mathbb{A}_{i,n}^+ \mathbb{A}_{i,n}^+ = 0, \tag{3}$$

reminiscent of fermions. The nilpotency condition arises from the fact that only one signal can be extracted from a given detector at any given time.

The above rules are not ad hoc but consequences of the basic definition (1) and the completeness of the signal states. It is possible to construct variants of the signal operators which behave more like fermionic operators, following the methodology of Jordan and Wigner (Jordan and Wigner 1928, Bjorken and Drell 1965). Under appropriate circumstances, it should be possible to use their methods to construct the equivalent of fermionic and bosonic signal field theories (Eakins and Jaroszkiewicz 2005)

The signal operators allow the construction of various *signal classes*, consisting of states created by a given number of distinct signal operators. The zero-signal class consists of one element only, namely the void state $|0, n)$. The one-signal class consists of the states of the form $\mathbb{A}_{i,n}^+|0, n)$, and there are exactly r_n such states. Likewise, the two-signal class consists of all states of the form $\mathbb{A}_{i,n}^+ \mathbb{A}_{j,n}^+|0, n)$, and there are $r_n!/2!(r_n - 2)!$ such states, and so on.

There are $r_n + 1$ distinct signal classes, and altogether they give the 2^{r_n} signal states which form the natural basis B_n for $\mathcal{H}_n$. An arbitrary labstate is a normalized linear combination of any of these basis states. In the applications to particle decays discussed in this paper, only one-signal states are needed. A one-signal state can sometimes describe what would correspond to a multi-particle state in SQM. What determines the interpretation of a labstate is the contextual information available to the observer as to what their elementary detectors mean. For example, the question *"has this particle decayed into a fireball consisting of five hundred particles?"* would require only one qubit, not five hundred.

A. Dynamics

It is clear from the approach being taken that SSQM avoids the assumptions of continuity which M&S referred to in their analysis. In particular, the Hilbert spaces involved are finite dimensional, reflecting the empirical fact that all experimentalist have to bin their data, even when it is assumed that there is a continuum of outcomes.

In SSQM, dynamics is described in terms of mappings of labstates from one Heisenberg net to its successor Heisenberg net. A *Born map* is defined here to be any map from one Hilbert space $\mathcal{H}$ to some other Hilbert space $\mathcal{H}'$ which preserves norm, i.e., if Ψ in $\mathcal{H}$ is mapped into $\mathfrak{B}(\Psi) \equiv \Psi'$ in $\mathcal{H}'$ by a Born map $\mathfrak{B}$, then

$$(\Psi', \Psi') = (\Psi, \Psi). \tag{4}$$

Born maps are essential here in order to preserve total probability (hence the terminology), but unfortunately, this is insufficient for quantum applications. Born maps are not necessarily linear, as can be seen from the construction of elementary examples. To go further, it is necessary to impose linearity.

A *semi-unitary operator* is defined to be a linear Born map, i.e., if $\Psi = \alpha\psi + \beta\phi$ for ψ, ϕ in $\mathcal{H}$ and α, β complex, then

$$U(\Psi) = \alpha U(\psi) + \beta U(\phi) \tag{5}$$

if U is semi-unitary. The imposition of linearity, which is an established feature of SQM, makes a powerful impact in SSQM. It can be readily shown that a semi-unitary operator U from $\mathcal{H}$ into $\mathcal{H}'$ exists if and only if $d \equiv \dim \mathcal{H} \leqslant d' \equiv \dim \mathcal{H}'$. Moreover, it can be proved that semi-unitarity implies that

$$U^+U = I, \tag{6}$$

where I is the identity for $\mathcal{H}$. From this, it follows immediately that a semi-unitary operator preserves inner products and not just norms, i.e., if $\Psi' \equiv U\Psi$ and $\Phi' \equiv U\Phi$, where Ψ and Φ are arbitrary elements of $\mathcal{H}$, then

$$(\Phi', \Psi') = (\Phi, \Psi) \tag{7}$$

if U is semi-unitary. If in fact $d < d'$ then necessarily

$$UU^+ \neq I', \tag{8}$$

where I' is the identity for $\mathcal{H}'$.

In SSQM, the dynamics is given in terms of a sequence of semi-unitary evolution operators $\mathbb{U}_{n+1,n}$ taking labstates in $\mathcal{H}_n$ to labstates in $\mathcal{H}_{n+1}$ and so on. Such operators satisfy the rule

$$\mathbb{U}_{n+1,n}^+ \mathbb{U}_{n+1,n} = \mathbb{I}_n. \tag{9}$$

Further details are given in (Jaroszkiewicz and Eakins, 2006)

III.   ONE SPECIES DECAYS

In this section, SSQM is used to describe the quantum physics of an unstable particle state X which can decay to a state Y. At all times total probability will be manifestly conserved.

Every run of an experiment to observe such a decay process will be assumed to start at time $t = 0$, at which time the observer knows that they have prepared an X state (to use the language of SQM). In SSQM, this is represented by the labstate $|\Psi, 0\rangle \equiv \mathbb{A}_{X,0}^+|0,0\rangle$, which is automatically normalized to unity.

By time 1, the labstate will have changed from $|\Psi, 0\rangle$ to some new labstate $|\Psi, 1\rangle$ of the form

$$|\Psi, 1\rangle = \alpha \mathbb{A}_{X,1}^+|0,1\rangle + \beta \mathbb{A}_{Y_1,1}^+|0,1\rangle, \qquad (10)$$

where the complex numbers α and β satisfy the semi-unitarity rule

$$|\alpha|^2 + |\beta|^2 = 1. \qquad (11)$$

The amplitude $\mathcal{A}(X, 1|X, 0)$ for the particle not to have decayed by time 1 is given by the rule

$$\mathcal{A}(X, 1|X, 0) \equiv \langle 0, 1|\mathbb{A}_{X,1}|\Psi, 1\rangle = \alpha \qquad (12)$$

whilst the amplitude $\mathcal{A}(Y, 1|X, 0)$ for the particle to have made the transition to state X by time 1 is given by

$$\mathcal{A}(Y, 1|X, 0) \equiv \langle 0, 1|\mathbb{A}_{Y_1,1}|\Psi, 1\rangle = \beta. \qquad (13)$$

Total probability is therefore conserved. Note that on the right hand side of (13) the label Y is itself labeled by a subscript, in this case the number 1, which is the time at which the amplitude is calculated for. It will be seen that the time at which a transition occurs is a crucial feature of the analysis, being directly related to the measurement issues discussed by M&S.

Such a process conserves signal class, so the dynamics can be discussed wholly in terms of the evolution of the signal operators rather than the labstates. For instance, evolution from 0 to 1 can be given in the form

$$\mathbb{A}_{X,0}^+ \rightarrow \mathbb{U}_{1,0} \mathbb{A}_{X,0}^+ \mathbb{U}_{1,0}^+ = \alpha \mathbb{A}_{X,1}^+ + \beta \mathbb{A}_{Y_1,1}^+, \qquad (14)$$

where $\mathbb{U}_{1,0}$ is a semi-unitary operator satisfying the rule

$$\mathbb{U}_{1,0}^+ \mathbb{U}_{1,0} = \mathbb{I}_0, \qquad (15)$$

with $\mathbb{I}_0$ being the identity for the initial Heisenberg net $\mathcal{H}_0 \equiv \mathcal{Q}_0^X$.

The above process involves a change in rank, since $\mathcal{H}_1 \equiv \mathcal{Q}_1^X \otimes \mathcal{Q}_1^{Y_1}$. Because $r_1 \equiv \dim \mathcal{H}_1 > r_0 \equiv \dim \mathcal{H}_0$, semi-unitarity of the evolution operator means that

$$\mathbb{U}_{1,0} \mathbb{U}_{1,0}^+ \neq \mathbb{I}_1. \qquad (16)$$

Unlike SQM, therefore, the SSQM approach to quantum dynamics places clear constraints on what is meant by time reversal. Any attempt to discuss time reversal has to focus on the apparatus involved. Moreover, any such experiment always takes place in the same direction of time as the rest of the universe.

The description of the next stage of the process, from time 1 to time 2, is more subtle and involves the concept of *null test* . Such a test is defined here as any quantum test which extracts no information from a given initial state. In SQM, this corresponds to passing some outcome of an apparatus through the same or equivalent apparatus, the net effect being to leave the state unchanged. For example, an electron emerging from a Stern-Gerlach apparatus S_0 in the spin-up state will pass through another Stern-Gerlach apparatus S_1 completely unscathed, provided the magnetization axis of S_1 is in the same direction as that of S_0. In SQM, a null test is modelled mathematically by the fact that an eigenstate of an operator is also an eigenstate of the square of that operator.

Considering the labstate of the above decay process at time 1, there are now two terms to consider. The first term in (14), $\alpha \mathbb{A}_{X,1}^+$, corresponding to *no decay* by time 1, can be regarded at this point as creating an initial X state which could now decay into a Y state or not, with the same characteristics as for the first stage of the run, i.e., between times 0 and 1. This assumes spatial and temporal homogeneity, a physically reasonable assumption which would need to be reconsidered in gravitational fields, which are taken to be absent here for simplicity. The second term, $\beta \mathbb{A}_{Y_1,1}^+$, corresponds to *decay having occurred during the first time interval*, which is regarded here as irreversible. Situations where the Y state can revert back to the X state are more complicated and of greater empirical interest, such in the ammonium maser and neutral Kaon decay, and these are discussed in later sections of this paper.

Assuming homogeneity, the next stage of the evolution is given by

$$\begin{aligned} \mathbb{U}_{2,1} \mathbb{A}_{X,1}^+ \mathbb{U}_{2,1}^+ &= \alpha \mathbb{A}_{X,2}^+ + \beta \mathbb{A}_{Y_2,2}^+, \\ \mathbb{U}_{2,1} \mathbb{A}_{Y_1,1}^+ \mathbb{U}_{2,1}^+ &= \mathbb{A}_{Y_1,2}^+. \end{aligned} \qquad (17)$$

The second equation is justified as follows. The decay term in (14), proportional to $\mathbb{A}_{Y_1,1}^+$ at time 1, corresponds to the possibility of detecting a decay product state at that time. Now there is nothing which requires this information to be extracted precisely at that time. The experimentalist could choose to delay information extraction until some later time, effectively placing the decay product observation "on hold". As stated above, this may be represented in SQM by passing a state through a null-test, which does not alter it. In SSQM this is represented by the second equation in (17). Essentially, quantum information about a decay is passed forwards in time until it is physically extracted.

The Heisenberg net $\mathcal{H}_2$ at time 2 has rank three, being the tensor product

$$\mathcal{H}_2 = \mathcal{Q}_2^X \otimes \mathcal{Q}_2^{Y_1} \otimes \mathcal{Q}_2^{Y_2}. \qquad (18)$$

Semi-unitary evolution from time zero to time 2 therefore gives

$$\begin{aligned} \mathbb{A}_{X,0}^+ \rightarrow \mathbb{U}_{2,1} \mathbb{U}_{1,0} \mathbb{A}_{X,0}^+ \mathbb{U}_{1,0}^+ \mathbb{U}_{2,1}^+ &= \alpha^2 \mathbb{A}_{X,2}^+ + \alpha\beta \mathbb{A}_{Y_2,2}^+ \\ &\quad + \beta \mathbb{A}_{Y_1,2}^+, \end{aligned} \qquad (19)$$

with the various probabilities being read off as the squared moduli of the corresponding terms.

It is clear that in (**??**) a space-time description with a specific arrow of time is being built up, with a memory of the change of rank of the Heisenberg net at time 1 being propagated forwards in time to time 2. This is represented by the contribution involving $\mathbb{A}_{Y_{1,2}}^{+}$, from a potential decay process which may have occurred by time 1, and this contributes to the overall labstate amplitude at time 2.

Subsequently the process continues in an analogous fashion, with the rank of the Heisenberg net increasing by one at each timestep. At time n the dynamics gives

$$\mathbb{A}_{X,0}^{+} \to \mathbb{U}_{n,0}\mathbb{A}_{X,0}^{+}\mathbb{U}_{n,0}^{+} = \alpha^{n}\mathbb{A}_{X,n}^{+} + \beta\sum_{k=1}^{n}\alpha^{k-1}\mathbb{A}_{Y_{k},n}^{+}, \tag{20}$$

where $\mathbb{U}_{n,0} \equiv \mathbb{U}_{n,n-1}\mathbb{U}_{n-1,n-2}\ldots\mathbb{U}_{1,0}$ is semi-unitary and satisfies the constraint

$$\mathbb{U}_{n,0}^{+}\mathbb{U}_{n,0} = \mathbb{I}_{0}. \tag{21}$$

From the above, the survival probability $\Pr(X,n|X,0)$ that the original state has *not* decayed can be easily read off and is found to be

$$\Pr(X,n|X,0) = |\alpha|^{2n}. \tag{22}$$

Provided $\beta \neq 0$, this probability appears to falls monotonically with increasing n, which corresponds to particle decay.

The discussion at this point calls for some care with limits, because here there arises the theoretical possibility of encountering the quantum Zeno effect, as discussed by M&S. In the following, it will be assumed that $|\alpha| < 1$, because $|\alpha| = 1$ corresponds to a stable particle, which is of no interest here.

Now consider the physics of the situation. The calculated probabilities should relate to the elapsed time t as used by the observer in the laboratory, which is not assumed here to be a continuous parameter. The temporal subscript n labelling successive stages corresponds to an elapsed time given by $t \equiv n\tau$, where τ is some well-defined time scale characteristic of the apparatus. In the sort of experiments relevant here, τ will be a very small fraction of a second, but certainly nowhere near the Planck time. Realistic measurement timescales, involving electromagnetic processes, would be in the $10^{-9} - 10^{-18}$ second range.

If now the transition amplitude α and τ are related by the rule

$$|\alpha|^{2} \equiv e^{-\Gamma\tau}, \tag{23}$$

where Γ is a characteristic inverse time, then the survival probability $P(t_{n})$ is given by

$$P(t) \equiv \Pr(X,n|X,0) = e^{-\Gamma t}, \tag{24}$$

which is the usual exponential decay formula. No imaginary term proportional to Γ in any supposed Hamiltonian or energy has been introduced in order to obtain this exponential decay law.

A subtlety arise here however. Expression (23) is equivalent to the assumption that $|\alpha|^{2}$ is an analytic function of τ with a Taylor expansion of the form

$$|\alpha|^{2} = 1 - \Gamma\tau + O(\tau)^{2}, \tag{25}$$

i.e., one with a non-zero linear term. Under such circumstances, the standard result $\lim_{n\to\infty}(1 - \frac{x}{n})^{n} = e^{-x}$ gives the exponential decay law. The possibility remains, however, that the dynamics is such that the linear term is zero, so that the appropriate expression is of the form

$$|\alpha|^{2} = 1 - \gamma\tau^{2} + O(\tau^{3}), \tag{26}$$

where γ is a positive constant (Bollinger et al. 1990). Then in the limit $n \to \infty$, where $n\tau \equiv t$ is held fixed, the result is given by

$$\lim_{n\to\infty, n\tau=t \text{ fixed}} \left(1 - \gamma\tau^{2} + O(\tau^{3})\right)^{n} = 1, \tag{27}$$

which gives rise to the quantum Zeno effect scenario.

To understand properly what is going on, it is necessary to appreciate that there are two competing limits being considered: one where a system is being observed over an increasing macroscopic laboratory time scale t, and another one where many observations are being taken in succession, separated by a microscopic time scale τ which is being brought as close to zero as possible. In each case, the limit cannot be achieved in the laboratory. The result is that in such experiments, the apparatus may play a decisive role in determining the measured outcomes. If the apparatus is such that (25) holds, then exponential decay will be observed. If on the other hand the apparatus behaves according to the rule (26), or any reasonable variant of it, then approximations to the quantum Zeno effect should be observed.

The scenario where apparatus plays a decisive role in observation was discussed by M&S, but not taken further, on the grounds that they saw no indication that the ""*observed lifetime*" of an unstable particle is not a property of the particle (and its Hamiltonian)" (Misra and Sudarshan 1977). It is probable that M&S included a reference a Hamiltonian because they recognized that, contrary to what they wanted to assert (i.e., that the dynamical evolution of a decaying particle is an intrinsic property of the particle alone), the external environment does play a role in observation. In fact, Hamiltonians change whenever the environment in which particles are situated is changed by the observer. For instance, if an electric field is switched on, the Hamiltonian associated with a charged particle changes. It is therefore incorrect to regard a Hamiltonian as an intrinsic property of a system under observation alone.

Our conclusion is that there is *every* indication that apparatus plays a role in the observed dynamics of particles.

A. Semi-unitary matrices

The above scenario can be discussed more efficiently in terms of semi-unitary matrices, an approach which will

be useful in the discussion of neutral Kaon decay given later.

A semi-unitary matrix M is a $r' \times r$ matrix mapping complex $r-$dimensional column vectors into complex $r'-$dimensional column vectors, such that

$$\mathsf{M}^+\mathsf{M} = \mathsf{I}_r, \qquad (28)$$

where I_r is the $r \times r$ identity matrix. No semi-unitary matrix exists if $r' < r$.

Now consider the X decay scenario discussed above. If the initial labstate is represented by the 1×1 matrix $\Psi_0 \equiv [1]$, then the action of $\mathbb{U}_{1,0}$ given by (10) may be represented by the semi-unitary matrix

$$\mathsf{U}_{1,0} \equiv \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \qquad (29)$$

Then the labstate at time 1 is represented by the 2×1 matrix Ψ_1 given by

$$\Psi_1 = \mathsf{U}_{1,0}\Psi_0 = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} [1] = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \qquad (30)$$

The two required transition amplitudes are just the various components of this vector.

Going further, the action of $\mathbb{U}_{2,0}$ is represented by the 3×2 semi-unitary matrix

$$\mathsf{U}_{2,1} = \begin{bmatrix} \alpha & 0 \\ \beta & 0 \\ 0 & 1 \end{bmatrix}, \qquad (31)$$

and so on for later times. For arbitrary $n > 1$, it is found that

$$\mathsf{U}_{n,n-1} = \begin{bmatrix} \alpha & 0 \\ \beta & 0 \\ 0 & \mathsf{I}_{n-1} \end{bmatrix}, \qquad (32)$$

where I_n is the $n \times n$ identity matrix. This gives

$$\Psi_n = \mathsf{U}_{n,n-1}\mathsf{U}_{n-1,n-2}\ldots\mathsf{U}_{1,0}\Psi_0 = \begin{bmatrix} \alpha^n \\ \beta\alpha^{n-1} \\ \vdots \\ \beta\alpha \\ \beta \end{bmatrix}. \qquad (33)$$

The squared modulus of the first component of this column vector gives the same survival probability $|\alpha|^{2n}$ as before. It is also relatively easy now to read off all the other probabilities and give a discrete time version of the P, Q and R functions discussed by M&S. Let P_n be the probability that an X state created at time 0 has made a transition to a Y state sometime during the temporal interval $[1, n]$, inclusive of the end times, let Q_n be the probability that an initially prepared X state has not made such a transition at any time during this interval, and let $R_{m,n}$ be the probability that an initially prepared X state had not made a transition between time 0 and m, and then made a transition sometime between $m+1$ and n inclusive (assuming $0 \leqslant m < n$). Then clearly

$$P_n + Q_n = 0, \qquad R_{m,n} = Q_m P_{n-m}. \qquad (34)$$

From the components of the Ψ_n, it is found that

$$P_n = 1 - |\alpha|^{2n}, \quad Q_n = |\alpha|^{2n}, \quad n = 0, 1, 2, \ldots, \qquad (35)$$

from which $R_{m,n}$ can be constructed.

Although this analysis gives results which look formally like the standard decay result, the scenario is equivalent to that discussed by M&S, namely, there is a constant questioning (or its discrete equivalent) by the apparatus as to whether decay has taken place or not. In this case the results are simple. In more complicated scenarios, such as Kaon decay, the results are more complicated.

IV. THE AMMONIUM SYSTEM

A successful application of SQM to particle physics was the explanation by Gell-Mann and Pais (Gell-Mann and Pais 1955) of the phenomenon of regeneration in neutral Kaon decays. In the standard calculation (Leighton, Feynman, and Sands 1966), a non-hermitian Hamiltonian is used to introduce the two decay parameters needed to describe the observations. In the next section it will be shown how SSQM readily reproduces the result of the Gell-Mann-Pais calculation whilst conserving total probability.

The analysis of the Kaon system is more complex than the single particle decay process discussed above, involving the interplay of two distinct neutral Kaons, the K^0 and its antiparticle, the $\bar{K}^0$. In order to understand the SSQM approach to the description of neutral Kaon decays, it will be helpful to review first how systems such as the ammonium molecule are discussed in SQM and in SSQM.

When translation and rotational symmetries are ignored, the ammonium molecule is described in SQM in terms of a superposition of two orthonormal states representing the two possible position states of the single nitrogen atom relative to the plane defined by the three hydrogen atoms. These two states form a basis for a two-dimensional Hilbert space describing the system. The Hamiltonian for the system is represented by the Hermitian matrix

$$\mathsf{H} = \begin{bmatrix} e & f \\ f^* & g \end{bmatrix}, \qquad (36)$$

where e and g are real and f can be complex. If the state of the molecule is represented at time t by the two-component wave-function

$$\Psi(t) \equiv \begin{bmatrix} \Psi_1(t) \\ \Psi_2(t) \end{bmatrix}, \qquad (37)$$

then the Schrödinger equation $i\hbar\partial_t\Psi(t) = \mathsf{H}\Psi(t)$ has solutions

$$\Psi_j(t) = A_j e^{-i\omega^+ t} + B_j e^{-i\omega^- t}, \quad j = 1, 2, \qquad (38)$$

where A_j and B_j are constants and $\omega^\pm = \frac{1}{2}\{e + g \pm \sqrt{4|f|^2 + (e-g)^2}\}$. This gives probability functions

which have oscillatory behaviour with a frequency given by the difference $\omega^+ - \omega^-$.

In the SSQM description, it will be assumed that there are two different states, X, Y, with signal operators $\mathbb{A}_{X,n}^+$, $\mathbb{A}_{Y,n}^+$ respectively, evolving according to the rule

$$
\begin{aligned}
\mathbb{U}_{n+1,n}\mathbb{A}_{X,n}^+|0,n) &= \{a\mathbb{A}_{X,n+1}^+ + b\mathbb{A}_{Y,n+1}^+\}|0,n+1), \\
\mathbb{U}_{n+1,n}\mathbb{A}_{Y,n}^+|0,n) &= \{c\mathbb{A}_{X,n+1}^+ + d\mathbb{A}_{Y,n+1}^+\}|0,n+1),
\end{aligned}
\tag{39}
$$

where $\mathbb{U}_{n+1,n}$ is a semi-unitary operator. Semi-unitarity requires the constraints

$$
|a|^2 + |b|^2 = |c|^2 + |d|^2 = 1, \quad a^*c + b^*d = 0. \tag{40}
$$

All other states will be disregarded on the basis that there are no dynamical channels between them and states X and Y. With a suitable choice of phases, $\mathbb{U}_{n+1,n}$ can be represented by the semi-unitary matrix

$$
\mathsf{U} = \begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix}, \tag{41}
$$

where an overall possible phase factor is ignored and a and b are as above. The eigenvalues $z^\pm$ of U are given by

$$
z^\pm = \frac{1}{2}\left\{ a + a^* \pm i\sqrt{4 - (a+a^*)^2} \right\}. \tag{42}
$$

These are complex conjugates of each other and have magnitude unity, so can be written in the form $z^\pm = \exp\{\pm i\theta\}$, where θ is real. Writing $a \equiv |a|e^{i\alpha}$, where α is real, then $\cos\theta = |a|\cos\alpha$. Now U can always be written in the form

$$
\mathsf{U} = \mathsf{V}\begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix}\mathsf{V}^+, \tag{43}
$$

where V is semi-unitary. Again, with a suitable choice of phases, V can be written in the form

$$
\mathsf{V} = \begin{bmatrix} u & -v^* \\ v & u^* \end{bmatrix}, \tag{44}
$$

where $|u|^2 + |v|^2 = 1$, and then it is found that

$$
|u|^2 e^{i\theta} + |v|^2 e^{-i\theta} = a, \quad u^*v(e^{i\theta} - e^{-i\theta}) = b. \tag{45}
$$

This gives

$$
\begin{aligned}
\mathsf{U}^n &= \mathsf{V}\begin{bmatrix} e^{in\theta} & 0 \\ 0 & e^{-in\theta} \end{bmatrix}\mathsf{V}^+ \\
&= \begin{bmatrix} |u|^2 e^{in\theta} + |v|^2 e^{-in\theta} & uv^*\{e^{in\theta} - e^{-in\theta}\} \\ u^*v\{e^{in\theta} - e^{-in\theta}\} & |u|^2 e^{-in\theta} + |v|^2 e^{in\theta} \end{bmatrix},
\end{aligned}
\tag{46}
$$

which leads to the dynamical rule

$$
\begin{aligned}
\mathbb{A}_{X,0}^+ &\to \{|u|^2 e^{in\theta} + |v|^2 e^{-in\theta}\}\mathbb{A}_{X,n}^+ \\
&\qquad + u^*v\{e^{in\theta} - e^{-in\theta}\}\mathbb{A}_{Y,n}^+, \\
\mathbb{A}_{Y,0}^+ &\to uv^*\{e^{in\theta} - e^{-in\theta}\}\mathbb{A}_{X,n}^+ \\
&\qquad + \{|u|^2 e^{-in\theta} + |v|^2 e^{in\theta}\}\mathbb{A}_{Y,n}^+.
\end{aligned}
\tag{47}
$$

Hence the probabilities are given by

$$
\begin{aligned}
\Pr(X,n|X,0) &= \Pr(Y,n|Y,0) \\
&= |u|^4 + |v|^4 + 2|u|^2|v|^2\cos(2n\theta), \\
\Pr(Y,n|X,0) &= \Pr(X,n|Y,0) \\
&= 4|u|^2|v|^2\sin^2(n\theta),
\end{aligned}
\tag{48}
$$

which agrees with the SQM expressions when $2n\theta = (\omega^+ - \omega^-)t$.

It was noted in (Bollinger et al. 1990) that a survival probability of the form $P(\tau) \sim 1 - \gamma\tau^2 + O(\tau^3)$ would be needed to make observations of the quantum Zeno effect viable. The above calculation of the ammonium survival probabilities is compatible with this, as can be seen from the expansion

$$
\begin{aligned}
\Pr(X,n|X,0) &= |u|^4 + |v|^4 + 2|u|^2|v|^2\cos(2n\theta) \\
&\sim 1 - 4|u|^2|v|^2 n^2\theta^2 + O\left(n^4\theta^4\right).
\end{aligned}
\tag{49}
$$

Therefore, it is expected that the quantum Zeno effect (or at least some behaviour analogous to it) should be observable in the ammonium system, provided the amplitudes involved have the required dependence on τ. As with the particle decays discussed above, it would be necessary to ensure that the two limits, $n \to \infty$, $\tau \to 0$, were carefully balanced, the point being that the laboratory protocol will play a decisive role in the outcome of the experiment.

V. KAON-TYPE DECAYS

More complex systems such as neutral Kaon decay are readily discussed in SSQM as follows. Consider three different particle states, X, Y and Z, making transitions between each other in the specific way described below. An important example occurring in particle physics involves the neutral Kaons, with X representing a K^0 meson, Y representing a $\bar{K}^0$ meson, and Z representing their various decay products. Kaon decay is remarkable for the phenomenon of regeneration, in which the Kaon survival probabilities fall and then rise with time. More recently, a similar phenomenon has been observed in B meson decay.

As before, attention can be focused on one-signal states states. The dynamics is described by the transition rules

$$
\begin{aligned}
\mathbb{A}_{X,n}^+ &\to \alpha\mathbb{A}_{X,n+1}^+ + \beta\mathbb{A}_{Y,n+1}^+ + \gamma\mathbb{A}_{X_{n+1},n+1}^+, \\
\mathbb{A}_{Y,n}^+ &\to u\mathbb{A}_{X,n+1}^+ + v\mathbb{A}_{Y,n+1}^+ + w\mathbb{A}_{Z_{n+1},n+1}^+, \\
\mathbb{A}_{Z_n,n}^+ &\to A_{Z_n,n+1}^+,
\end{aligned}
\tag{50}
$$

where semi-unitarity requires the transition coefficients to satisfy the constraints

$$
\begin{aligned}
|\alpha|^2 + |\beta|^2 + |\gamma|^2 &= 1, \\
|u|^2 + |v|^2 + |w|^2 &= 1, \\
\alpha^*u + \beta^*v + \gamma^*w &= 0.
\end{aligned}
\tag{51}
$$

The above process is a combination of the decay and oscillation processes discussed previously.

By inspection of (50), it can be seen that there are no transitions from Z states to either X or Y states. Therefore, once a Z state is created, it remains a Z state. There is an irreversible flow from the X and Y states, so these eventually disappear. Before that occurs however, there will be back-and-forth transitions between the X and Y states which give rise to the phenomenon of regeneration.

In actual Kaon decay experiments, pure K^0 states can be prepared via the strong interaction process $\pi^- + p \to K^0 + \Lambda$, whilst pure $\bar{K}^0$ states can be prepared via the process $\pi^+ + p \to K^+ + \bar{K}^0 + p$. In our notation, these preparations correspond to initial labstates $\mathbb{A}_{X,0}^+|0,0)$ and $\mathbb{A}_{Y,0}^+|0,0)$ respectively. In practice, superpositions of K^0 and $\bar{K}^0$ states may be difficult to prepare directly, but the analysis of Gell-Mann and Pais shows that such states can be obtained indirectly (Gell-Mann and Pais 1955). Therefore, labstates corresponding to X and Y superpositions are physically meaningful and will be used in the following analysis.

Consider an initial labstate of the form

$$|\Psi, 0) \equiv \left\{ a\mathbb{A}_{X,0}^+ + b\mathbb{A}_{Y,0}^+ \right\} |0,0), \qquad (52)$$

where $|a|^2 + |b|^2 = 1$. Matrix methods are appropriate here. The dynamics of the system will be discussed in terms of the initial column vector

$$\Psi_0 \equiv \begin{bmatrix} a \\ b \end{bmatrix}, \qquad (53)$$

which is equivalent to the statement that each run of the experiment starts with the rank-two Heisenberg net $\mathcal{H}_0 \equiv \mathcal{Q}_0^X \otimes \mathcal{Q}_0^Y$. The dynamical rules (50) map labstates in $\mathcal{H}_0$ into $\mathcal{H}_1 \equiv \mathcal{Q}_1^X \otimes \mathcal{Q}_1^Y \otimes \mathcal{Q}_1^{Z_1}$, so there is a change of rank from two to three. The transition is represented by the semi-unitary matrix

$$\mathsf{U}_{1,0} \equiv \begin{bmatrix} \alpha & u \\ \beta & v \\ \gamma & w \end{bmatrix}. \qquad (54)$$

More generally, we may write

$$\mathsf{U}_{n+1,n} \equiv \begin{bmatrix} \alpha & u & 0 \\ \beta & v & 0 \\ \gamma & w & 0 \\ 0 & 0 & \mathsf{I}_n \end{bmatrix}, \quad n > 0, \qquad (55)$$

where I_n is the $n \times n$ identity. The Heisenberg net at time n has rank $n+2$ and changes to one of rank $n+3$ over the next time step.

If the state at time t is represented by a column vector Ψ_n with $n+2$ components, then we may write

$$\Psi_n = \mathsf{U}_{n,n-1}\mathsf{U}_{n-1,n-2}\ldots\mathsf{U}_{2,1}\mathsf{U}_{2,0}\Psi_0. \qquad (56)$$

Overall probability is conserved, because the semi-unitarity of the transition operators $\mathsf{U}_{i+1,i}$ guarantees that

$$\Psi_n^+ \Psi_n = \Psi_0^+ \Psi_0. \qquad (57)$$

Once again, the key to unravelling the dynamics is to use linearity, which is guaranteed by the use of semi-unitary evolution operators. Suppose the state Ψ_n at time n is represented by

$$\Psi_n = \begin{bmatrix} x_n \\ y_n \\ z_{n,n} \\ \vdots \\ z_{1,n} \end{bmatrix}, \qquad (58)$$

where the components x_n and y_n are such that

$$x_n = \lambda^n x_0, \ y_n = \lambda^n y_0, \qquad (59)$$

where λ is some complex number to be determined. Such states will be referred to as eigenmodes. They are not eigenstates of any physical operator, but their first two components, x_n and y_n behave as if they were. Then the dynamics gives the following relations:

$$\begin{aligned} x_{n+1} &= \alpha x_n + y_n = \lambda x_n, \\ x_{n+1} &= \beta x_n + v y_n = \lambda y_n, \\ z_{n+1,n+1} &= \gamma x_n + w y_n. \end{aligned} \qquad (60)$$

Experimentalists will be interested only in survival probabilities for the X and Y states, so the dynamics of Z states will be ignored here, i.e., the behaviour of the components $z_{k,n}$ for $k < n$ will not be discussed.

It will be seen from the above that λ is an eigenvalue of the matrix

$$\mathsf{M} \equiv \begin{bmatrix} \alpha & u \\ \beta & v \end{bmatrix}, \qquad (61)$$

which means that in principle there are two solutions, λ^+ and λ^-, for the eigenmode values, given by

$$\lambda^\pm = \frac{\alpha + v \pm \sqrt{(\alpha - v)^2 + 4\beta u}}{2}. \qquad (62)$$

It is expected that these will not be mutual complex conjugates in actual experiments, because if they were, the analysis could not explain observed Kaon physics. Therefore, the coefficients α, β, u and v will be such that the above two eigenmode values are complex and of different magnitude and phase, giving rise to two decay channels with different lifetimes, as happens in neutral Kaon decay. In the SQM analysis of neutral Kaon decays, Gell-Mann and Pais described the neutral Kaons as superpositions of two hypothetical particles known as K_1^0 and K_2^0, which are CP eigenstates and have different decay lifetimes (Gell-Mann and Pais 1955). The K_1^0 decays to a two pion state with a lifetime of about 0.9×10^{-10} seconds whilst the K_2^0 decays to a three pion state with a lifetime of about 0.5×10^{-7} seconds.

Semi-unitarity guarantees that

$$|x_{n+1}|^2 + |y_{n+1}|^2 + |z_{n+1,n+1}|^2 = |x_n|^2 + |y_n|^2, \quad (63)$$

and so it can be deduced that

$$|\lambda|^2 = 1 - \frac{|z_{n+1,n+1}|^2}{|x_n|^2 + |y_n|^2} < 1, \qquad n = 0, 1, 2, \ldots \quad (64)$$

From this and the conditions

$$x_n = \lambda^n x_0, \quad y_n = \lambda^n y_0, \tag{65}$$

the eigenmode values can be written in the form

$$\lambda_1 = r_1 e^{i\theta_1}, \quad \lambda_1 = r_2 e^{i\theta_2}, \tag{66}$$

where $r_1 < 1$, $r_2 < 1$ and θ_1 and θ_2 are real. The eigenmodes at time $t = 0$ corresponding to λ_1 and λ_2 will be denoted by $\Lambda_{1,0}$ and $\Lambda_{2,0}$ respectively, i.e.

$$\Lambda_{1,0} = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}, \Lambda_{2,0} = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}, \tag{67}$$

and then the evolution rules give

$$\Lambda_{1,n} = \begin{bmatrix} \lambda_1^n a_1 \\ \lambda_1^n b_1 \\ c_{n,n} \\ \vdots \\ c_{1,n} \end{bmatrix}, \quad \Lambda_{2,n} = \begin{bmatrix} \lambda_2^n a_2 \\ \lambda_1^n b_2 \\ d_{n,n} \\ \vdots \\ d_{1,n} \end{bmatrix}, \tag{68}$$

where the coefficients $\{c_{k,n}\}, \{d_{k,n}\}$ can be determined from the dynamics. The initial modes $\Lambda_{1,0}$ and $\Lambda_{2,0}$ are linearly independent provided λ_1 and λ_2 are different. Given that, then any initial labstate Ψ_0 can be expressed uniquely as a normalized linear combination of $\Lambda_{1,0}$ and $\Lambda_{2,0}$, i.e.,

$$\Psi_0 = \mu_1 \Lambda_{1,0} + \mu_2 \Lambda_{2,0}, \tag{69}$$

for some coefficients μ_1 and μ_2. This is the analogue of the decompositions

$$|K^0\rangle = \frac{1}{\sqrt{2}} \left\{ |K_1^0\rangle + |K_2^0\rangle \right\}, \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}} \left\{ |K_1^0\rangle - |K_2^0\rangle \right\} \tag{70}$$

in the Gell-Mann-Pais approach.

From this, the amplitude $\mathcal{A}(X, n|\Psi, 0)$ to find an X signal at time n is given by

$$\mathcal{A}(X, n|\Psi, 0) = \mu_1 a_1 \lambda_1^n + \mu_2 a_2 \lambda_2^n, \tag{71}$$

so that the survival probability for X is given by

$$\begin{aligned} \Pr(X, n|\Psi, 0) = & |\mu_1|^2 |a_1|^2 r_1^{2n} + |\mu_2|^2 |a_2|^2 r_2^{2n} \\ & + 2 r_1^n r_2^n \operatorname{Re} \left\{ \mu_1^* \mu_2 a_1^* a_2 e^{-in(\theta_1 - \theta_2)} \right\}, \end{aligned} \tag{72}$$

and similarly for $\Pr(Y, n|\Psi, 0)$.

There is scope here for various limits to be considered, as discussed in the single channel decay analysis, such that either particle decay is seen or the quantum Zeno effect appears to hold over limited time spans. If we write

$$r_1^n \equiv e^{-\Gamma_1 t/2}, \quad r_{2^n} \equiv e^{-\Gamma_2 t/2} \tag{73}$$

where $t \equiv n\tau$ and Γ_1, Γ_2 correspond to long and short lifetime decay parameters respectively, then the various constants can always be adjusted to get agreement with the standard Kaon survival intensity functions

$$\begin{aligned} I\left(K^0\right) &= \tfrac{1}{4}(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-(\Gamma_1+\Gamma_2)t/2} \cos \Delta mc^2 t/\hbar), \\ I\left(\bar{K}^0\right) &= \tfrac{1}{4}(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1+\Gamma_2)t/2} \cos \Delta mc^2 t/\hbar) \end{aligned} \tag{74}$$

for pure K^0 decays. Here Δm is proportional to the proposed mass difference between the hypothetical K_1^0 and K_2^0 "particles", which are each CP eigenstates and are supposed to have CP conserving decay channels. From the SSQM approach, such objects do not exist. Instead, they are simply manifestations of different possible superpositions of K^0 and $\bar{K}^0$ labstates, which are physically realizable via the strong interactions given above.

VI. CONCLUDING REMARKS

In this paper, it has been shown how signal-state quantum mechanics gives an instrumentalist description of particle decays and the quantum Zeno effect, consistent with Heisenberg's approach to quantum mechanics. It provides an alternative description of quantum processes with a novel interpretation of quantum wave-functions and avoids any reliance on the metaphysical concepts of system under observation and continuous time. Instead of thinking about elementary particles as strange, nonclassical objects which can sometimes appear to be waves and sometimes particles, we can choose instead to think only of how laboratory apparatus responds to physical manipulation. This is surely a more correct way to discuss physics, rather than in terms of classically motivated and suspect objectification.

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[1] In earlier papers, our formalism was referred to as "system-free" quantum mechanics, but this has been changed here to avoid any undue association with metaphysics.

[2] SSQM has the scope to discuss those situations where an observer is uncertain about the apparatus itself, rather than about the labstate. The latter possibility would be represented by a mixed labstate, corresponding to a mixed state in SQM. The former possibility has no conventional analogue in SQM. There is no room to comment further on this point here.